FORCES ON A POROUS SPHERE SPINNING IN A FLUID FLOW

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Exact expressions are found for the drag (modified Stokes force) and the lift (modified Magnus force) on a porous sphere spinning slowly in a viscous fluid flowing slowly and uniformly past it.

1. Statement of the Problem. Suppose a porous sphere of radius R rotating with a constant angular velocity ω_l is located at the origin of coordinates in a uniform flow V_i = const, where VR/ν and $\omega R^2/\nu < 1$. The general expression for the force on the sphere due to the flow can be obtained from the requirement of covariance. For convenience we introduce in place of the angular velocity pseudovector ω_l the true antisymmetric tensor $\beta_{ik} = \varepsilon_{ikl}\omega_l$ dual to it. Assuming slow motions, VR/ν and $\omega R^2/\nu < 1$, we have

$$F_i = \varphi V_i + \theta\, \beta_{ik} \cdot V_k + 0(V_i,\ \beta_{ik} V_k), \tag{1.1}$$

where φ and θ are constant scalars depending on the viscosity and density of the fluid, the size of the sphere, and the volume and surface permeability coefficients of the porous sphere.

The term φV_i represents a correction to the Stokes resistance force from the porosity, and $\theta\beta_{ik} V_k$ is a correction to the lift. The lift is a bilinear (cross) effect, and its evaluation requires a solution of the problem which takes account of the cross terms in the nonlinear terms of the Navier–Stokes equation and because of the assumption of slow motions does not require taking account of terms containing squares of the angular velocity and the ambient velocity. Thus, from the very beginning we can exclude from consideration centrifugal forces inside the porous spinning sphere.

We note that in nonstationary flow $[V_i = V_i(t),\ \beta_{ik} = \beta_{ik}(t)]$ the force will not be a function but a functional of $V_i(t)$ and $\beta_{ik}(t)$ and their time derivatives. In particular, the functional will contain the aftereffect integral. In this case φ and θ of (1.1) will not be constants, but will depend on the time and the integrands of the functionals. Since the character of the time dependence of φ and θ cannot be established from covariant or dimensional arguments, the functional analog of (1.1) for nonstationary motion becomes useless.

We consider steady-state motion. To determine the explicit form of the force (1.1), i.e., the coefficients φ and θ, it is necessary to know the perturbed velocity and pressure fields resulting from the presence of the spinning sphere. Their calculation requires solving two problems. The first or outer problem requires solving the Navier–Stokes equation

$$v_k \frac{\partial v_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\Delta v_i, \quad \frac{\partial v_i}{\partial x_i} = 0, \quad r > R, \tag{1.2}$$

$$v_i \to V_i \quad \text{as} \quad r \to \infty,$$

and the second or inner problem requires solving the Darcy equation for permeation flow of fluid in the porous sphere,

$$\Delta\Pi = 0, \quad Q_i = -\frac{k}{\eta}\frac{\partial \Pi}{\partial x_i}, \quad r < R. \tag{1.3}$$

As was shown above, the centrifugal force must be omitted in the approximation considered.

Here v_i and p are the velocity and pressure of the fluid outside the sphere, Q_i and Π are the permeation velocity and the pressure of the fluid inside the sphere, k is the permeability of the sphere material, and η is the dynamic viscosity of the fluid.

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Three boundary conditions are set on the surface of the porous sphere: the equality of the normal components of the outer and permeation velocities

$$n_i v_i = n_i Q_i, \quad r = R; \tag{1.4}$$

the equality of the outer normal stress and the pressure inside the sphere

$$p - \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right) n_i n_k = \Pi, \quad r = R \tag{1.5}$$

and, finally, the boundary condition connecting the gradient of the tangential component of the outer velocity and the difference in tangential components of the outer and permeation velocities

$$\frac{\partial v_m}{\partial r} - n_m \frac{\partial}{\partial r}(n_p v_p) = \frac{\alpha}{k^{1/2}}[v_m - Q_m - (\vec{\omega} \times \vec{R})_m], \quad r = R. \tag{1.6}$$

Boundary condition (1.6) is proposed on the basis of experiments in [1] and is theoretically justified in [2]. Its physical meaning becomes clear if we note that at the boundary of a porous body the velocity of the fluid changes from v outside the body to Q inside the body over a certain characteristic distance depending on the porous properties of the body. Since the permeability k has the dimensions of the square of a length, there is only one characteristic distance in a porous body, in addition to the size of the body itself, and it is proportional to $\sqrt{k}$. Thus, the change in velocity in the region near the surface occurs over a distance $\Delta r \sim \sqrt{k}$, and, consequently, we have for the velocity gradient in this region $\Delta v/\partial r \sim (v - Q)/\sqrt{k}$ or $\partial v/\partial r = \alpha\,[(v - Q)/k^{1/2}]$. Taking account of the curvature of the sphere leads to Eq. (1.6). Thus, the coefficient α introduced in (1.6) reflects the surface properties of the porous body just as the coefficient k reflects its volume properties, and both coefficients must be determined experimentally for each porous material. The solution of the problem is constructed as an expansion in dimensionless small parameters proportional, respectively, to V_i and β_{ik}, although dimensionless variables will not be used explicitly later on. The first approximation, linear in V_i and β_{ik}, does not contain their product and therefore does not take account of the nonlinear terms in the Navier–Stokes equation. In the second approximation there are retained everywhere, including the nonlinear term $v_k(\partial v_i/\partial x_k)$ of the Navier–Stokes equation, only the cross terms proportional to $\beta_{ik}V_m$, i.e., those responsible for the lift.

2. First (Linear) Approximation. The first-approximation problem differs from the complete problem (1.2)-(1.6) only in that we have instead of (1.2)

$$0 = -\frac{\partial p^{(1)}}{\partial x_i} + \eta \cdot \Delta v_i^{(1)}, \quad \frac{\partial v_i^{(1)}}{\partial x_i} = 0, \quad v_i^{(1)} \to V_i \quad \text{as} \quad r \to \infty. \tag{2.1}$$

Applying the divergence operation to the Navier–Stokes equation (2.1), we obtain

$$\Delta p^{(1)} = 0. \tag{2.2}$$

The solution of (2.2) in which the perturbation of the pressure field by the sphere vanishes at infinity has the form

$$p^{(1)} = \eta\left[\frac{a}{r} + a_i \frac{\partial}{\partial x_i}\left(\frac{1}{r}\right) + a_{ik}\frac{\partial^2}{\partial x_i \partial x_k}\left(\frac{1}{r}\right) + \dots\right].$$

The tensors a, a_i, $a_{ik}, \dots$ are constants and must depend linearly on the constant tensors V_i and β_{ik} in the first approximation which we are now constructing. From the requirement of covariance, and noting that $\beta_{ik} = -\beta_{ki}$, we obtain $a = 0$ and $a = AV_i$. All tensors of higher rank vanish. Thus,

$$p^{(1)} = \eta A V_m \frac{\partial}{\partial x_m}\left(\frac{1}{r}\right). \tag{2.3}$$

Substituting the expression obtained for the pressure into (2.1), we have for the velocity

$$\Delta v_i^{(1)} = \frac{1}{\eta}\frac{\partial p^{(1)}}{\partial x_i} = AV_m \frac{\partial^2}{\partial x_i \partial x_m}\left(\frac{1}{r}\right). \tag{2.4}$$

Using the fact that $\Delta r = 2/r$, we find the particular solution of Eq. (2.4) $v_{i\,\text{partic}}^{(1)} = (AV_m/2)(\partial^2/\partial x_i \partial x_m) r$. The solution of the homogeneous equation (2.4) satisfying the condition $v_i^{(1)} \to V_i$ as $r \to \infty$ has the form: $v_{i\,\text{homo}}^{(1)} = V_i + (c_i/r) + c_{ik}(\partial/\partial x_k)(1/r) + c_{ikl}(\partial^2/\partial x_k \partial x_l)(1/r) + \dots$. Like the coefficients a, a_i, $a_{ik}, \dots$, the coefficients c_i, $c_{ik}, \dots$ must also be linear covariant combinations of the tensors V_i and β_{ik}, which uniquely determines their form: $c_i = cV_i$, $c_{ik} = D\beta_{ik}$, $c_{ikl} = \varepsilon\delta_{ik}V_l$. All tensors of higher rank vanish. Writing the expression for the velocity and satisfying the equation of continuity we obtain for the velocity

$$v_i^{(1)} = V_i + \frac{AV_m}{2}\frac{\partial^2}{\partial x_i \partial x_m}(r) - \frac{AV_i}{r} + D\beta_{ik}\frac{\partial}{\partial x_k}\left(\frac{1}{r}\right) + \varepsilon V_k \frac{\partial^2}{\partial x_i \partial x_k}\left(\frac{1}{r}\right). \tag{2.5}$$

We turn to the inner problem. Substituting (2.3) and (2.5) into (1.3) and (1.5) we obtain for the permeation flow in the sphere the Dirichlet problem

$$\Delta\Pi^{(1)}=0,\quad \Pi^{(1)}(R)=\eta V_m n_m\left(-\frac{3A}{R^2}+\frac{12\varepsilon}{R^4}\right). \tag{2.6}$$

We seek a series solution of (2.6) in the form $\Pi^{(1)}=\gamma+\gamma_i x_i+\gamma_{ik}x_ix_k+\gamma_{ikl}x_ix_kx_l+\dots$

From boundary condition (2.6) it follows that $\gamma=\gamma_{ik}=\gamma_{ikl}=0$, $\gamma_i=\eta V_i\,[-(3A/R^3)+(12\varepsilon/R^5)]$. The equation $\Delta\Pi^{(1)}=0$ is satisfied automatically in this case so that we have inside the sphere

$$\Pi^{(1)}=\eta V_i\left(-\frac{3A}{R^3}+\frac{12\varepsilon}{R^5}\right)x_i;\quad Q_i=-\frac{k}{\eta}\frac{\partial\Pi}{\partial x_i}=kV_i\left(\frac{3A}{R^3}-\frac{12\varepsilon}{R^5}\right).$$

Now satisfying the remaining two boundary conditions (1.4) and (1.6) we obtain a system of equations whose solution gives the coefficients

$$A=\frac{3+3\dfrac{k^{1/2}}{\alpha R}}{\left(2+3\dfrac{k}{R^2}\right)+\dfrac{k^{1/2}}{\alpha R}\left(4+15\dfrac{k}{R^2}\right)}R,\quad D=\frac{R^3}{1+2\dfrac{k^{1/2}}{\alpha R}}, \tag{2.7}$$

$$\varepsilon=\frac{\dfrac{1}{2}-\dfrac{1}{2}\dfrac{k^{1/2}}{\alpha R}}{\left(2+3\dfrac{k}{R^2}\right)+\dfrac{k^{1/2}}{\alpha R}\left(4+15\dfrac{k}{R^2}\right)}R^3.$$

3. Second Approximation for the Outer Problem. We write the equation for the outer problem in the second approximation, denoting second-approximation quantities by the superscript (2),

$$v_k^{(1)}\frac{\partial v_i^{(1)}}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p^{(2)}}{\partial x_i}+\nu\Delta v_i^{(2)},\quad \frac{\partial v_i^{(2)}}{\partial x_i}=0,\quad r>R. \tag{3.1}$$

Substituting into (3.1) the expression for $v_i^{(1)}$ from (2.5) and, in accordance with the problem posed, selecting in the left-hand Eq. (3.1) only the cross terms proportional to $\beta_{ik}V_m$, we obtain

$$\Delta v_i^{(2)}=\frac{1}{\eta}\frac{\partial p^{(2)}}{\partial x_i}+\frac{D\beta_{il}V_l}{\nu}\left(-\frac{1}{r^3}+\frac{A}{2r^4}+\frac{\varepsilon}{r^6}\right)+\frac{D\beta_{il}n_lV_mn_m}{\nu}\left(\frac{3}{r^3}-\frac{2A}{r^4}\right)+\frac{DV_k\beta_{km}n_mn_i}{\nu}\left(\frac{A}{2r^4}-\frac{3\varepsilon}{r^6}\right). \tag{3.2}$$

Applying the divergence operation to (3.2), we obtain the equation for $p^{(2)}$,

$$\Delta p^{(2)}=\rho D\beta_{im}V_m\frac{\partial}{\partial x_i}\left(\frac{A}{4r^4}-\frac{3\varepsilon}{r^6}\right). \tag{3.3}$$

A particular solution of (3.3) is

$$p^{(2)}_{\text{partic}}=\rho D\beta_{im}V_m\frac{\partial}{\partial x_i}\left(\frac{A}{8r^2}-\frac{\varepsilon}{4r^4}\right). \tag{3.4}$$

We seek the solution of the homogeneous equation in the form

$$p^{(2)}_{\text{homo}}=\frac{s}{r}+s_i\frac{\partial}{\partial x_i}\left(\frac{1}{r}\right)+s_{ik}\frac{\partial^2}{\partial x_i\partial x_k}\left(\frac{1}{r}\right)+\dots \tag{3.5}$$

From the fact that the constant tensors s, s_i, s_{ik}, ... must be linear covariant combinations of the tensor $\beta_{ik}V_m$ or its contraction, it follows uniquely that

$$s=0,\quad s_i=\rho fDV_m\beta_{mi}. \tag{3.6}$$

All tensors of higher rank are equal to zero (ρD is introduced for uniformity of notation in subsequent formulas). Thus, from (3.4)-(3.6)

$$p^{(2)}=\rho DV_m\beta_{mk}\frac{\partial}{\partial x_k}\left(\frac{f}{r}-\frac{A}{8r^2}+\frac{\varepsilon}{4r^4}\right). \tag{3.7}$$

Substituting (3.7) into (3.2) and making transformations to simplify the integration, we obtain the equation

$$\Delta v_i^{(2)}=\frac{D\beta_{im}V_m}{\nu}\left(-\frac{A}{8r^4}+\frac{3\varepsilon}{2r^6}\right)+\frac{DV_m\beta_{mk}}{\nu}\frac{\partial^2}{\partial x_i\partial x_k}\left(\frac{f}{r}-\frac{A}{16r^2}+\frac{\varepsilon}{8r^4}\right)+\frac{DV_l\beta_{im}}{\nu}\frac{\partial^2}{\partial x_m\partial x_l}\left(\frac{1}{r}-\frac{A}{4r^2}\right). \tag{3.8}$$

We write a particular solution of this equation:

$$v^{(2)}_{i\,\text{partic}}=\frac{D\beta_{im}V_m}{\nu}\left(-\frac{A}{16r^2}+\frac{\varepsilon}{8r^4}\right)+\frac{DV_m\beta_{mk}}{\nu}\frac{\partial^2}{\partial x_i\partial x_k}\left(\frac{fr}{2}-\right.$$
$$\left.-\frac{A\ln r}{16}+\frac{\varepsilon}{16r^2}\right)+\frac{DV_l\beta_{im}}{\nu}\frac{\partial^2}{\partial x_m\partial x_l}\left(\frac{r}{2}-\frac{A}{4}-\ln r\right)$$

and the solution of the homogeneous equation (3.8): $v_{i\,\mathrm{homo}}^{(2)} = (\tau_i/r) + \tau_{ik}(\partial/\partial x_k)(1/r) + \tau_{ikl}(\partial/\partial x_k \partial x_l)(1/r) + \ldots$, where the constant tensors τ_i, τ_{ik}, ... are proportional to the tensor $\beta_{ik}V_m$ or its contraction $\beta_{ik}V_k$. Satisfying the covariance requirement we obtain: $\tau_i = (D/\nu)\,\Phi V_m \beta_{mi}$; $\tau_{ik} = 0$, $\tau_{ikl} = (D/\nu)\psi V_k \beta_i + (D/\nu)\tau\delta_{ik}V_m\beta_{ml}$. Tensors of higher rank vanish (D/ν is introduced for uniformity of notation in subsequent formulas). Satisfying the equation of continuity we obtain the following expression for the outer velocity in the second approximation:

$$v_i^{(2)} = \frac{D\beta_{im}V_m}{\nu}\left(\frac{f-1}{2r} - \frac{A}{4r^2} + \frac{\tau-\psi}{r^3} + \frac{\varepsilon}{4r^4}\right) + \frac{DV_m\beta_{mk}n_in_k}{\nu}\left(-\frac{f}{2r} + \frac{A}{8r^2} + \frac{3\tau}{r^3} + \frac{\varepsilon}{2r^4}\right) + \frac{DV_l\beta_{im}n_mn_l}{\nu}\left(-\frac{1}{2r} + \frac{A}{2r^2} + \frac{3\psi}{r^3}\right). \tag{3.9}$$

4. Second Approximation for the Inner Problem. Expressions for the Drag and Lift. We write the equations for the inner problem and the boundary conditions on the surface of the sphere in the second approximation:

$$\Delta\Pi^{(2)} = 0, \quad Q_i^{(2)} = -\frac{k}{\eta}\frac{\partial\Pi^{(2)}}{\partial x_i}, \quad r < R, \tag{4.1}$$

$$n_iv_i^{(2)} = n_iQ_i^{(2)}, \quad r = R, \tag{4.2}$$

$$p^{(2)} - \eta\left(\frac{\partial v_i^{(2)}}{\partial x_k} + \frac{\partial v_k^{(2)}}{\partial x_i}\right)n_in_k = \Pi^{(2)}, \quad r = R, \tag{4.3}$$

$$\frac{\partial v_k^{(2)}}{\partial r} - n_k\frac{\partial}{\partial r}\left(n_mv_m^{(2)}\right) = \frac{\alpha}{k^{1/2}}\left(v_k^{(2)} - Q_k^{(2)}\right), \quad r = R. \tag{4.4}$$

Condition (4.4) does not contain the angular velocity explicitly, since (4.4) represents boundary condition (1.6) written for second-order quantities in V and ω. The term $[\alpha/k^{1/2}](\vec{\omega}\times\vec{R})$ from (1.6) is of first order and has been completely taken into account in the first approximation.

Substituting (3.7) and (3.9) into (4.1) and (4.3) we obtain a Dirichlet problem for the second approximation in the permeation flow:

$$\Delta\Pi^{(2)} = 0; \quad \Pi^{(2)}(R) = \rho DV_m\beta_{mk}n_k\left(-\frac{3f+1}{R^2} + \frac{7A}{4R^3} + \frac{12\tau+6\psi}{R^4} + \frac{\varepsilon}{R^5}\right).$$

We seek a series solution of this equation in the form $\Pi^{(2)} = g + g_ix_i + g_{ik}x_ix_k + \ldots$. It follows from boundary condition (4.5) that $g = g_{ik} = g_{ikl} = \ldots 0$.

$$g_k = \rho DV_m\beta_{mk}\left(-\frac{3f+1}{R^3} + \frac{7A}{4R^4} + \frac{12\tau+6\psi}{R^5} + \frac{\varepsilon}{R^6}\right).$$

The equation $\Delta\Pi^{(2)} = 0$ is satisfied automatically in this case. Thus, for the inner problem we have in the second approximation

$$\Pi^{(2)} = \rho DV_m\beta_{mk}\left(-\frac{3f+1}{R^3} + \frac{7A}{4R^4} + \frac{12\tau+6\psi}{R^5} + \frac{\varepsilon}{R^6}\right)x_k, \tag{4.6}$$

$$Q_i^{(2)} = -\frac{k}{\eta}\frac{\partial\Pi^{(2)}}{\partial x_i} = \frac{k}{\nu}DV_m\beta_{mi}\left(\frac{3f-1}{R^3} - \frac{7A}{4R_4} - \frac{12\tau+6\psi}{R^5} - \frac{\varepsilon}{R^6}\right).$$

It now remains to satisfy conditions (4.2) and (4.4). Substituting (3.9) and (4.6) into them, we obtain a system of equations whose solution gives the coefficients

$$f = \frac{\left(-8 - \frac{39}{2}\frac{k}{R^2}\right) + \frac{\alpha R}{k^{1/2}}\left(-\frac{32}{3} + \frac{59}{2}\frac{k}{R^2} + 180\frac{k^2}{R^4}\right) + \frac{\alpha^2R^2}{k}\left(-\frac{14}{3} + \frac{63}{2}\frac{k}{R^2} + 96\frac{k^2}{R^4}\right) + \frac{\alpha^3R^3}{k^{3/2}}\left(-\frac{2}{3} + \frac{13}{2}\frac{k}{R^2} + 12\frac{k^2}{R^4}\right)}{\left(32 + 240\frac{k}{R^2} + 450\frac{k^2}{R^4}\right) + \frac{\alpha R}{k^{1/2}}\left(\frac{128}{3} + 248\frac{k}{R^2} + 330\frac{k^2}{R^4}\right) + \frac{\alpha^2R^2}{k}\left(\frac{56}{3} + 80\frac{k}{R^2} + 78\frac{k^2}{R^4}\right) + \frac{\alpha^3R^3}{k^{3/2}}\left(\frac{8}{3} + 8\frac{k}{R^2} + 6\frac{k^2}{R^4}\right)}, \tag{4.7}$$

$$\tau = \frac{\left(\frac{4}{9} - \frac{161}{12}\frac{k}{R^2} - 50\frac{k^2}{R^4}\right) + \frac{\alpha R}{k^{1/2}}\left(-\frac{19}{9} + \frac{11k}{R^2}\right) + \frac{\alpha^2R^2}{k}\left(-\frac{35}{18} - \frac{k}{4R^2} + \frac{12k^2}{R^4}\right) + \frac{\alpha^3R^3}{k^{3/2}}\left(-\frac{7}{18} + \frac{2k}{3R^2} + 2\frac{k^2}{R^4}\right)}{\left(32 + 240\frac{k}{R^2} + 450\frac{k^2}{R^4}\right) + \frac{\alpha R}{k^{1/2}}\left(\frac{128}{3} + 248\frac{k}{R^2} + 330\frac{k^2}{R^4}\right) + \frac{\alpha^2R^2}{k}\left(\frac{56}{3} + 80\frac{k}{R^2} + 78\frac{k^2}{R^4}\right) + \frac{\alpha^3R^3}{k^{3/2}}\left(\frac{8}{3} + 8\frac{k}{R^2} + 6\frac{k^2}{R^4}\right)}\cdot R^2,$$

$$\psi = \frac{\left(-1+\frac{15}{2}\frac{k}{R^2}\right)+\frac{\alpha R}{k^{1/2}}\left(-\frac{3}{2}+9\frac{k}{R^2}\right)+\frac{\alpha^2R^2}{k}\left(-\frac{1}{2}+\frac{3}{2}\frac{k}{R^2}\right)}{\left(36+135\frac{k}{R^2}\right)+\frac{\alpha R}{k^{1/2}}\left(30+72\frac{k}{R^2}\right)+\frac{\alpha^2R^2}{k}\left(6+9\frac{k}{R^2}\right)}\cdot R^2.$$

This completes the determination of the pressure and velocity fields to the accuracy necessary to determine the forces acting on the porous sphere. Substituting into $F_i = \oint\limits_{r=R} \left[-p\delta_{ik} + \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)\right] ds_k$ the expressions $p = p^{(1)} + p^{(2)}$ and $v = v^{(1)} + v^{(2)}$ found above in which the coefficients A, D, ε, f, τ, and ψ are determined by Eqs. (2.7) and (4.7), and integrating, we obtain the result

$$\vec{F} = s_1 6\pi R\eta\vec{V} + s_2\cdot\pi\rho R^3(\vec{V}\times\vec{\omega}), \text{ where} \tag{4.8}$$

$$s_1 = \frac{1+\frac{k^{1/2}}{\alpha R}}{\left(1+\frac{3}{2}\frac{k}{R^2}\right)+\frac{k^{1/2}}{\alpha R}\left(2+\frac{15}{2}\frac{k}{R^2}\right)},$$

$$s_2 = \frac{\left(1-\frac{47}{4}\frac{k}{R^2}-21\frac{k^2}{R^4}\right)+\frac{k^{1/2}}{\alpha R}\left(7-\frac{269}{4}\frac{k}{R^2}-183\frac{k^2}{R^4}\right)+\frac{k}{\alpha^2R^2}\left(16-\frac{425}{4}\frac{k}{R^2}-435\frac{k^2}{R^4}\right)+\frac{k^{3/2}}{\alpha^3R^3}\left(12-\frac{123}{4}\frac{k}{R^2}-225\frac{k^2}{R^4}\right)}{\left(1+3\frac{k}{R^2}+\frac{9}{4}\frac{k^2}{R^4}\right)+\frac{k^{1/2}}{\alpha R}\left(9+36\frac{k}{R^2}+\frac{135}{4}\frac{k^2}{R^4}\right)+\frac{k}{\alpha^2R^2}\times\left(30+153\frac{k}{R^2}+\frac{729}{4}\frac{k^2}{R^4}\right)+\frac{k^{3/2}}{\alpha^3R^3}\left(44+276\frac{k}{R^2}+\frac{1665}{4}\frac{k^2}{R^4}\right)+\frac{k^2}{\alpha^4R^4}\left(24+\frac{105k}{R^2}+\frac{675k^2}{2R^4}\right)}. \tag{4.9}$$

The first term in (4.8) represents a modified Stokes force acting on the porous sphere, and the second term is the lift. In the limit $k=0$ (impermeable sphere) $s_1 = s_2 = 1$, and (4.8) goes over into the corresponding formula obtained in [3, 4] for an impermeable sphere. An expression for the Stokes drag experienced by a porous sphere in a uniform flow was found recently in [5], where, however, instead of the boundary condition (1.6) there was imposed the incorrect (cf., [1, 2]) requirement of the equality of the tangential components of the outer and permeation velocities at the surface of the porous sphere. In addition, we note that the result obtained in [5] can be obtained formally from (4.8) and (4.9), by setting $\alpha = \infty$ and $\omega = 0$. A nonrotating sphere was considered in [5].

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